Comparative Analysis of Spatial Diversity Gain against Correlation and Power Imbalance

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Abstract

The report presents an analytical framework for analyzing two-branch diversity systems for a Rayleigh fading channel. In many cases the fading received at both branches (ie, a two antenna element system) is correlated because of the proximity of the antenna elements to each other an analytical expression for the probability density function of the signal-to-noise ratio at the output of a two-branch maximal ratio and selection diversity system is developed in this report. The two branches are assumed to be Rayleigh fading, correlated, as well as of unequal signal-to-noise ratios. The diversity gain for selection, maximal ratio, and equal gain combining for the 10% probability level is presented as a function of power imbalance and correlation between branches for a two-branch Rayleigh diversity system. The result obtained from measurement of diversity gain against correlation and power imbalance shows that the optimum combining technique among the three combiners is the maximal ratio combining.

INTRODUCTION

A radio channel is characterized by multipath signal between the transmitter and the receiver. Multipath interference arises due to reflection, diffraction and scattering of signal in the process of transmission. This behavior adversely affects the quality of wireless communication signal.

Multipath reception is the combination of the original direct line of sight (L.O.S) signal plus the duplicate wave fronts that result from reflection of the wave off obstacles as it travels between the transmitter and the receiver (see fig.1). When a radio frequency signal is transmitted towards the receiver, the general behavior of the signal is to grow wider (or spread out) as it travels further. On its way, the radio frequency signal encounters objects that reflect, refract and interfere with the signal. When such signal is reflected off an obstacle, multiple wave fronts are created. As a result new duplicate wave fronts reach the receiver. Under this condition multiple propagations occur when radio frequency signals take different paths from source to destination. A part of the signal travels direct to the destination while another bounces off an obstruction to travel a longer distance to the destination. However multipath distortion occurs due to radio frequency interference when a radio signal has more than one path to travel between the transmitter and the receiver.

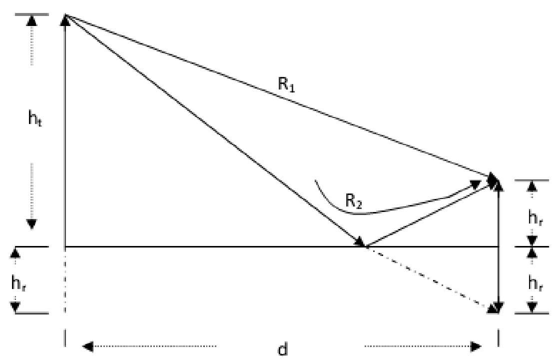

Fig 1: Effect of multipath on signal path

Most wireless communication systems, however, are low power and do not have the dynamic range available to counter the effects introduced by the propagation environment (distortion). An increase in reliability in multipath fading environment without increasing transmit power can be efficiently achieved using a receive antenna diversity system. Multiple antennas at the receiver have been used successfully in operational system to diminish the variance of local signal strength fluctuation by using the signals on all antenna elements to reduce the incidence of severe signal degradation that occur during a fade. Using several antennas increases the probability that one or more of the elements will receive signals with adequate signal strength. Reducing the occurrence of fades improves the overall reliability of the received information and therefore allows for greater coverage distance.

In present cellular mobile radio communications (824-894MHZ), the use of multiple antennas was almost exclusively limited to base station where a sufficiently large area was available to place several bulky antennas. It is well known that the size of antenna is directly proportional to its operating wavelength (Warren et al, 2003). The increase in communication frequencies, as a consequence, was accompanied by a reduction in size of the antenna elements. In addition at personal communication services (PCS) frequencies (1850-1900MHZ) or higher, it has become feasible to have multiple antennas not only at the base station but also at the handset.

Diversity is an effective method for increasing the received signal-to-noise ratio in a flat fading environment. The mobile radio channel varies with time and at times a receiver might receive a signal that is indistinguishable from the noise. Diversity is meant to provide the receiver with alternate paths to the transmitted signal to ensure the signal is reliably received. This paper will examine a two-branch diversity systems with focus on receive antenna diversity.

The signal envelope received by an antenna in a multiple reflective non-dispersive medium can, in theory, be modeled as Rayleigh fading in an environment where multipath components have equally distributed amplitudes. Measurements performed have established that it is not uncommon to find Rayleigh channels in blocked line of sight indoor and outdoor propagation. This paper presents a theoretical approach to predict the performance of a two-branch diversity system in these types of channels. The motivation for this analysis is to evaluate the performance of a two-antenna element handheld receiver. Due to size constraints, antenna elements on a handset are closely spaced (less than a wavelength, λ).

The performance of any diversity system also depends on the combining technique used to merge the signals received by the antenna elements. Among the most popular combining schemes are selection, equal gain and maximal-ratio. Some combining techniques outperform others under certain conditions and implementation issues usually determine which method is preferred. The results presented in this paper are meant to quantify the achievable gain of a two-branch diversity system which combines a pair of correlated and unbalanced channels. The theory developed in this paper assumes that both received signals undergo Rayleigh fading.

2.7 Diversity Combining Techniques

There are several schemes for connecting or using diversity branches to improve system performance. The main techniques in use are *selection*, *equal gain*, and *maximal ratio combining*. This write up focuses on the analysis of the three combining techniques. When using selection diversity, the receiver monitors the signal-to-noise ratio of all branches, selects and uses the information from the branch with the largest SNR. Equal gain combining requires the receiver to coherently sum the signals received through all channels in order to increase the available signal-to-noise ratio at the receiver. The most effective of these three is maximal ratio combining because at its output it presents the receiver with a signal-to-noise ratio that is the direct sum of all individual SNRs in the branches (Win et al, 1999). One of the drawbacks of maximal ratio combining is that the signal level and noise power at each branch need to be correctly estimated for all instances in time. This is fairly unrealistic in a practical system but the gains of this combining technique can be approached even if the required receiver estimations are not perfect for each branch.

2.11 Rayleigh Fading Channels

Clarke (Warren et al, 2003) proposed a model where the signal received by an antenna element in a static environment from a constant wave transmitter can be modeled as a sum of a total of N multipath components as follows

$$E_z = \sum_{n=1}^{N} A_n \cos(\omega_c t + \phi_n) \qquad 2.33$$

$$E_z = \sum_{n=1}^{N} A_n \cos\phi_n \cos\omega_c t - \sum_{n=1}^{N} A_n \sin\phi_n \sin\omega_c t$$

$$E_z = K_I \cos\omega_c t - K_Q \sin\omega_c t$$

$$K_I = \sum_{n=1}^{N} A_n \cos\phi_n \qquad 2.34$$

$$K_Q = \sum_{n=1}^{N} A_n \sin\phi_n$$

where A_n is the amplitude of the individual multipath components and ϕ_n is the phase.

Figure 2.9 depicts an example scenario in which those multipath components, five in number, are arriving at the receiver from different angles in the azimuth plane.

The signals are assumed to arrive in the azimuth plane where the receiving antenna has an omni directional pattern. To simplify this analysis requires expanding the expression for the received signal and grouping the factors multiplying $\cos w_c t$ and $\sin w_c t$ into separate components as was done in (2.34). K_I corresponds to the factors multiplying the in-phase carrier ($\cos w_c t$), while K_Q is the summation of terms multiplying the quadrature carrier ($\sin w_c t$).

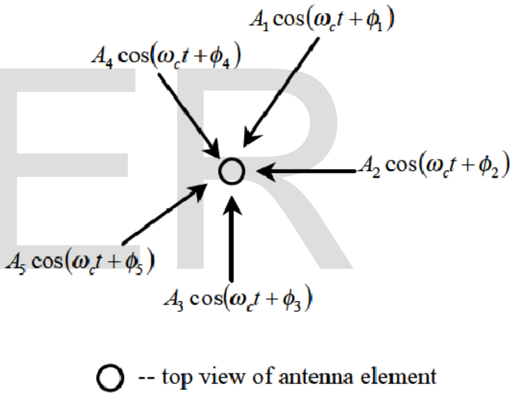

Figure 2.9: Five multipath components arriving at an antenna element in the azimuth plane

The statistical distributions of the phases and amplitudes and their interrelationships determine the distribution of the received signal. The phase, ϕ_n, depends on a combination of factors including the distance and medium in which the signal travels as well as on the phase alterations that are introduced by objects obstructing the propagation path.

The phase of a wave changes by 180^o (or π) for every half wavelength the signal travels (which corresponds to 7.5 cm at 2 GHz). As long as the distance covered by a traveling wave from the transmitter to the receiver is several wavelengths and/or its path is blocked by obstructions that introduce random phases, the phase of an impinging multipath component can take on any value and hence will be distributed. The correlation between K_I and K_Q can be investigated from the following equations;

$$E[K_I K_Q] = \sum_{n=1}^{N}\sum_{m=1}^{N} E[A_n A_m] \iint_{\phi_n \phi_m} \cos\phi_n \sin\phi_m f_{\Phi_n \Phi_m}(\phi_n, \phi_m) d\phi_n d\phi_m$$
$$E[K_I K_Q] = \sum_{n=1}^{N}\sum_{m=1}^{N} E[A_n A_m] E[\cos\phi_n \sin\phi_m] \quad 2.35$$

Concentrating on the expected values involving ϕ_n and ϕ_m the following results can be

established:

$$E[\cos\phi_n] = \int_0^{2\pi} \cos\phi_n f_{\Phi_n}(\phi_n) d\phi_n = \int_0^{2\pi} \cos\phi_n \frac{1}{2\pi} d\phi_1 = 0 \quad 2.36$$
$$E[\cos\phi_n \sin\phi_n] = \int_0^{2\pi} \cos\phi_n \sin\phi_n f_{\Phi_n}(\phi_n) d\phi_n = \int_0^{2\pi} \cos\phi_n \sin\phi_n \frac{1}{2\pi} d\phi_n = 0$$

By substituting the results from (2.36) into the last equation of (2.35) it is concluded that the in-phase component K_I and the quadrature component K_Q are uncorrelated or

$$E[K_I K_Q] = 0 \quad 2.37$$

The purpose of this analysis is to find the distribution of the received envelope by the antenna element. This analysis is simplified by introducing two variables H_n and T_n which are defined as

$$H_n = A_n \cos\phi_n$$
$$T_n = A_n \sin\phi_n \quad 2.38$$

which are the in-phase and quadrature components of each individual multipath

components. K_I and K_Q can be rewritten as

$$K_I = \sum_{n=1}^{N} H_n$$
$$K_Q = \sum_{n=1}^{N} T_n \quad 2.39$$

Which undergoes some process of evaluations necessary to give a summation of N independent random variables to create a new signal whose probability density function (pdf) can be computed by convolving the pdf s of all random variables. 2.40

$$f_{K_I}(K_I) = f_{H_1}(H_1) * f_{H_2}(H_2) * \cdots * f_{H_N}(H_N)$$
$$f_{K_Q}(K_Q) = f_{T_1}(T_1) * f_{T_2}(T_2) * \cdots * f_{T_N}(T_N)$$

$$E[K_I] = E[K_Q] = \sum_{n=1}^{N} E[H_n] = \sum_{n=1}^{N} E[T_n] = 0$$
$$E[K_I^2] = E[K_Q^2] = \sum_{n=1}^{N} E[H_n^2] = \sum_{n=1}^{N} E[T_n^2] = \frac{1}{2}\sum_{n=1}^{N} E[A_n^2] = \sigma^2 \quad 2.41$$

where the * operator represents convolution. As long as the variance of any individual component of A_n is much smaller than the sum of the variances of all the other components, or

$$E[A_n^2] << \sum_{n=1}^{N} E[A_n^2] \quad 2.42$$

for all n not all the random variables H_n and T_n have to be identically distributed to produce a signal with a Gaussian distribution.

From the previous discussion

$$E_z = K_I \cos\varpi_c t - K_Q \sin\varpi_c t \quad 2.43$$

In an environment with multipath where all the arriving signals are identically distributed in amplitude and the number of these impinging waves (N) is large, the signals K_I and K_Q will be Gaussian distributed with zero mean and equal variance. Mathematically, K_I and K_Q, have a probability density function given by

$$f_{K_I}(k_I) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{k_I^2}{2\sigma^2}}$$
$$f_{K_Q}(k_Q) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{k_Q^2}{2\sigma^2}} \qquad 2.44$$

where σ^2 is the variance of K_I and of K_Q.

In the third condition, when both branches are completely correlated, the correlation will become unity. That is, ρ = 1. As was stated before, when the branches are perfectly correlated, the received envelopes peak and fade at the same time. The pdf of m, which is a scaled version of the variable r, for perfectly correlated branches becomes (2.50).

$$f_M(m) = \frac{m}{(\sigma_1^2 + \sigma_2^2)} e^{-\frac{m^2}{2(\sigma_1^2+\sigma_2^2)}} \quad m \geq 0 \text{ and } \rho = \rho_P = 1 \qquad 2.50$$

The above equations describe the probability density function of the variable m at the output of the maximal ratio combiner for all combinations of envelope correlation and average powers (see fig3.7 and 4.8).

Furthermore, the distribution of the output SNRv after maximal ratio combining as a function of noise power N ($f_{NM}(Nm)$) can be found from $f_M(m)$ by applying the following change of variables

$$m_N = SNR_{V_M} = \sqrt{SNR_{p_M}} = \frac{1}{\sqrt{N}}\sqrt{r_1^2 + r_2^2} = \frac{1}{\sqrt{N}} m \qquad 2.51$$

The variables Nm and m are related by the multiplicative constant 1/√N.

The diversity gain for the 10% level after two-branch selection diversity is given below as;

$$G_D(10\%) = 5.71 e^{-0.87\rho - 0.16 D_M (dB)} \qquad 2.56$$

Below is the equation that shows the diversity gain of a two-branch maximal ratio diversity system as a function of envelope correlation and power imbalance at the 10% level;

$$G_D(10\%) = 7.14 e^{-0.59\rho - 0.11 D_M (dB)} \qquad 2.57$$

Equation below describes the diversity gain of a two-branch equal gain diversity system as a function of envelope correlation and power imbalance at the 10% level;

$$G_D(10\%) = -8.98 + 15.22 e^{-0.20\rho - 0.04 D_M (dB)} \qquad 2.58$$

Results and Discussion

Equations 2.49, 2.48, and 2.50 describe the probability density function of the variable m at the output of the maximal ratio combiner for all combinations of envelope correlation (ρ) and average branch powers (σ). The MATLAB programme showing graphical representations of the three probability density functions at the various correlation levels are given in figures 4.6, 4.7, and 4.8 of this chapter. Fig 4.6 describes the pdf of m at the output of maximal-ratio combiner at correlation point of 0.2, 0.4, 0.6, 0.8, which corresponds to what obtains within the range of 0< ρ <1. The whole graphical representation of the pdf (of fig. 4.6, 4.7 and 4.8) buttresses the point that there exists high probability of event when the branch signals are fully uncorrelated. While the probability of achieving reliable reception diminishes as the correlation of the branch signals approach unity.

Fig 4.7 describes the pdf at the output of MRC when there is zero correlation and equal branch power.

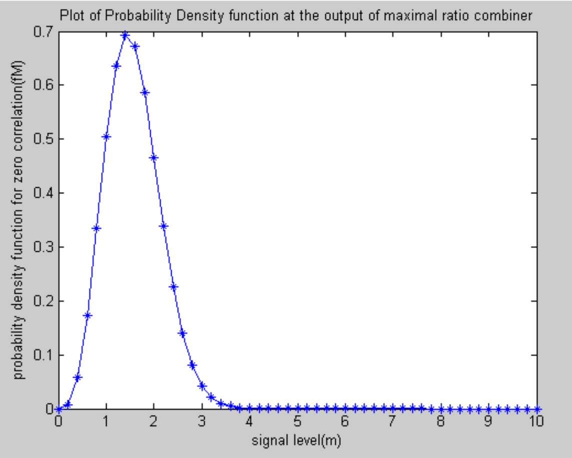

Fig.4.7: Plot of Probability Density function at the output of maximal ratio combiner (ρ = 0).

The equation that gives rise to the graph is valid for uncorrelated signal that have equal branch power.

Fig 4.8 is the plot of pdf of m at the output of MRC at unity correlation. It should be noted that at unity correlation, there is the tendency that branch signals peak and fade at the same time which may result to signal interruption at some instances.

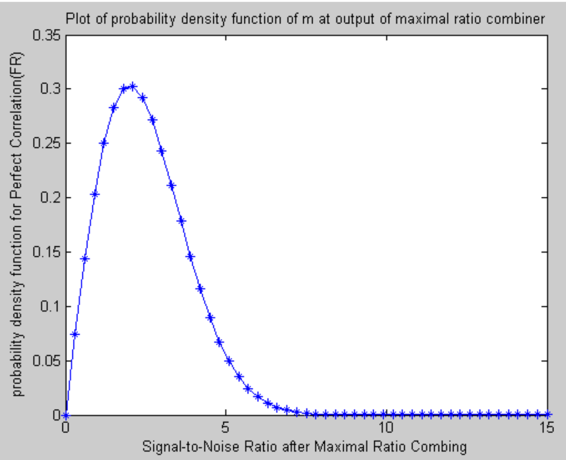

Fig 4.8: Plot of probability density function of m at output of maximal ratio combiner for unity correlation.

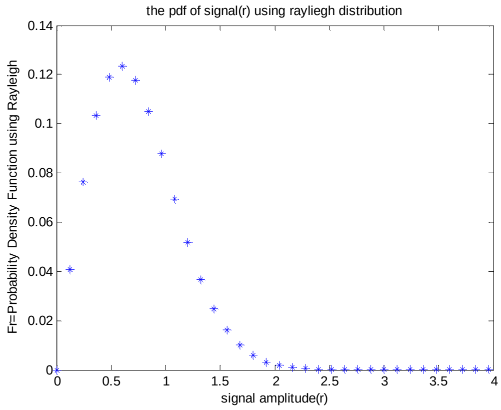

Fig4.9: Plot of probability density function of the variable r.

There is the likelihood for obtaining steady and reliable information (branch signals) at the output of MRC when correlation is below unity and more-so when branch signals are completely uncorrelated. It is deduced from fig 4.10 that there exists equal distribution of average branch signals at various level of correlation between zero and unity (0< ρ <1). Fig 4.11 shows a scanty distribution of average signal power (σ) at the lower threshold level of the CDF. Under perfect correlation, there is scanty distribution of signals at the lower level and clusters of signals at the upper signal level as shown in fig 4.12 of the CDF.

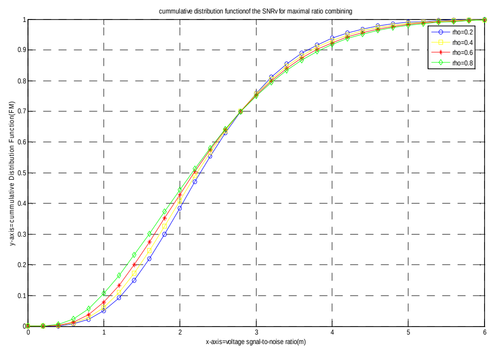

Fig 4.10: Plot of cumulative distribution function of m at output of maximal ratio combiner. (for ρ<1).

This graph is realized for all possible combinations of average branch power (σ_1, σ_2) and and when correlation (ρ) ranges between zero and unity.

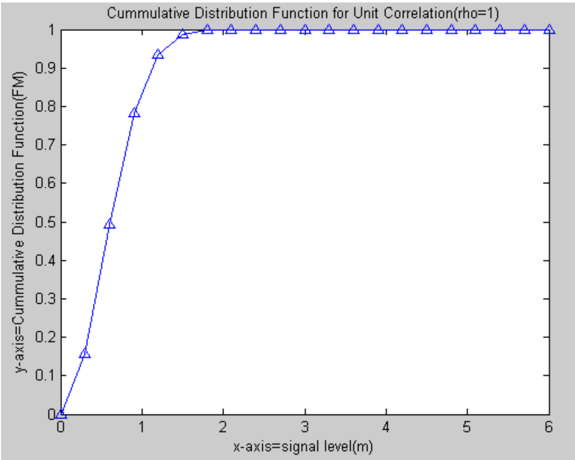

Fig: 4.11 Plot of Cumulative Distribution Function for Unity Correlation.

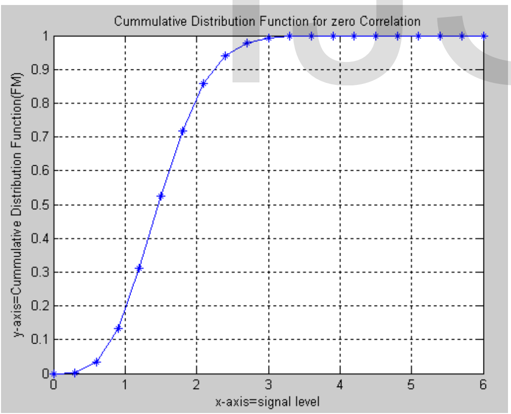

Fig: 4.12 Plot of Cumulative Distribution Function for Zero Correlation.

The situation here in this graph occurs when both average branch powers are equal ($\sigma_1 = \sigma_2 = \sigma$) and completely uncorrelated.

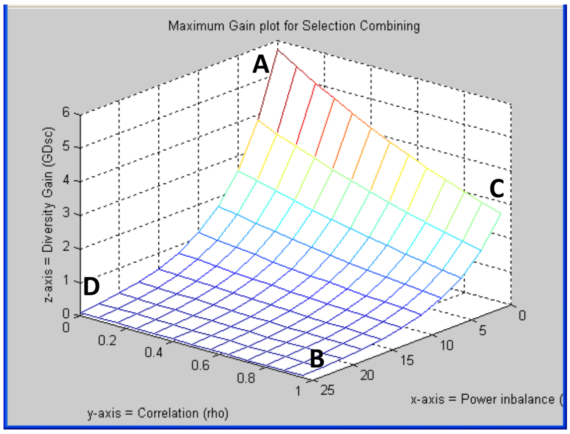

Fig4.13 Maximum Gain plot for Selection Combining.

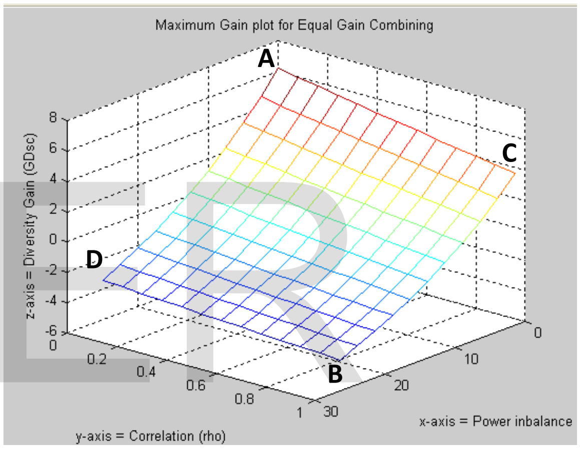

Fig:4.14 Maximum Gain plot for Equal Gain Combining.

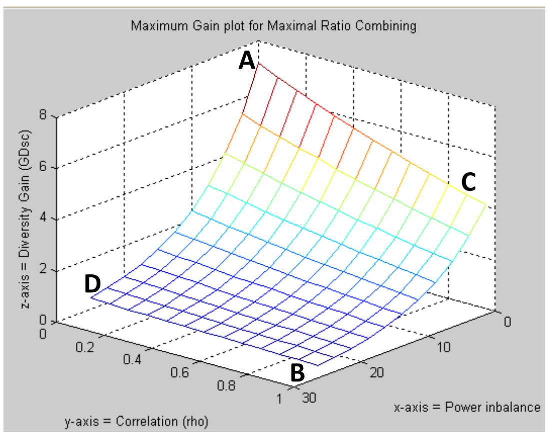

Fig: 4.15 Maximum Gain plot for Maximal Ratio Combining.

Table 1: Comparison of the relation in terms of correlation (ρ), power imbalance (D_M) and Diversity gain (G_D) for the three diversity systems applied in this thesis.

Scheme		SC	EGC	MRC
Difference in signal level (power Imbalance) D_M X-axis	A	0	0	0
	B	25	25	25
	C	0	0	0
	D	22.5	25	25
Correlation (ρ) (y-axis)	A	0	0	0
	B	0.88	0.96	0.96
	C	0.96	0.96	0.96
	D	0	0	0
G_D dB Z-axis	A	5.7	6.24	7.14
	B	0.05	-4.35	0.25
	C	0.64	3.58	4.05
	D	0.104	-3.38	0.456

There are eleven power imbalance values and thirteen correlation values which combine individually to produce the diversity gain values at their respective points as shown by the graph of fig:4.13 for a selection combiner. At point A which has Dm1 value of zero and ρ value of zero, the diversity gain is 5.7100. At power imbalance value of 25 and zero correlation, diversity gain is 0.1046 at point D. When correlation level shifts from zero to 0.08 value and power imbalance still remain at zero point, the diversity gain moves down to 5.3560. Also, at the point of power imbalance of 25 with correlation remaining at 0.08, the diversity gain moves to 0.0981. The move continues until it reaches the thirteenth correlation value marked B and C. The selected values at point marked C shows Dm1 = 0,

ρ = 0.99, and GDsc = 2.6491. Also at point marked B,

Dm1 = 25, ρ = 0.99, while GDsc = 0.0485.

The result obtained at points A, B, C, D, at the corners of the diversity gain plots of the three diversity schemes shows that at the point of very high power imbalance and correlation, maximal-ratio combining obtains the highest diversity gain as can be seen at points A and C. Under zero correlation and zero power imbalance at the point marked A, the selection combiner gain point is 5.7, equal gain combiner gain point is 6.24, while the maximal ratio combiner has 7.14 gain point. Looking at the overall gain of the three combining schemes, maximal-ratio combining acquires the highest gain and so, out performs the others.

CONCLUSION

The report presented a theoretical approach to analyzing a two-branch diversity system in a Rayleigh channel. In a Rayleigh fading channel, a single antenna can experience large fluctuations in received signal levels over small distances (when compared to **λ).** Diversity is a very docile solution to increase the reliability of the received information without increasing the transmit power. The motivation was to analyze the performance of a two antenna handheld unit. Due to physical size constraints of the unit, antennas are in close proximity of each other. The close proximity or the use of different antennas could cause both received signals to be correlated and /or have branches with different average powers. These two factors affect the performance of the diversity system. The effectiveness of the diversity also depends on the combining method that is being used.

From the combining methods presented in this report, maximal ratio combining performs the best. Selecting the better performer between selection and equal gain is not as straightforward. When both antennas have matched average powers, equal gain combining on the average is more effective. As the difference between average powers becomes larger, there is a point when selection diversity outperforms equal gain combining on the average as well as the instantaneous level.

This report presents an expression for the probability density function of the signal-to-noise ratio at the output of a two-branch selection and maximal ratio combining system for a Rayleigh fading environment. Both Rayleigh branches can potentially be correlated and have unequal average branch powers. Most of the work has only treated uncorrelated branch signals.

Performance of such a system, as can be seen from the gain plot at the 10% level, depends greatly on the correlation and average power difference between the branches. Table 4.14 has a collection of data at points A,B,C,and D of the diversity gain plot and shows the relationship of the three diversity schemes. The computed diversity gains for the two-branch Rayleigh system match very closely what Turkmani et al,1999 discovered by performing extensive measurements for a wide range of channels.

Transmit diversity has become increasingly an alternative to receive diversity systems for commercial purposes. Under the channel estimation, the same gain can be achieved through transmit diversity as with receive diversity systems.

The goal of diversity combining is to improve the received signal-to-noise ratio for all time. Though this report considers only two-branch diversity system, there is a high achievable gain by using more branches. As more and more branches are added to a diversity system, eventually flat fading can be completely mitigated.

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